Naslin polynomial method and multiresonant (a.k.a. muliti-resonant) current controllers?

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The controller and the plant (the open-loop transfer function)

$$G_{\mathrm{open}}(s) = \left(k_p + \frac{sk_1}{s^2+a} + \frac{sk_2}{s^2+b} + \frac{sk_3}{s^2+c} \right) \frac{1}{sL+R}$$

where

$$a = \omega_1^2,\ b = \omega_2^2,\ c = \omega_2^2$$

Let us reorganize this to get the characteristic polynomial of the closed-loop system

$$G_{\mathrm{open}}(s) = \frac{k_p(s^2+a)(s^2+b)(s^2+c) + sk_1(s^2+b)(s^2+c) + sk_2(s^2+a)(s^2+c) + sk_3(s^2+a)(s^2+b)}{(s^2+a)(s^2+b)(s^2+c)(sL+R)}$$

This gives

$$D_{\mathrm{closed}}(s) = (s^2+a)(s^2+b)(s^2+c)(sL+R) + k_p(s^2+a)(s^2+b)(s^2+c) + \\ + sk_1(s^2+b)(s^2+c) + sk_2(s^2+a)(s^2+c) + sk_3(s^2+a)(s^2+b)$$

Naslin polynomial

$$P_{\mathrm{Naslin}}(s) = a_0 \left(\frac{s^7\tau^7}{\alpha^{21}} + \frac{s^6\tau^6}{\alpha^{15}} + \frac{s^5\tau^5}{\alpha^{10}} + \frac{s^4\tau^4}{\alpha^6} + \frac{s^3\tau^3}{\alpha^3} + \frac{s^2\tau^2}{\alpha} + s\tau + 1 \right)$$

Just compare coefficients to get

$$k_p \underbrace{abc}_{c_0} + Rabc \stackrel{s^0}{=} a_0 \Leftrightarrow k_p = \frac{a_0}{c_0} - R$$

$$L \stackrel{s^7}{=} a_0 \frac{\tau^7}{\alpha^{21}} \Leftrightarrow a_0 = \frac{\alpha^{21} L}{\tau^7}$$

$$k_p + R \stackrel{s^6}{=} a_0 \frac{\tau^6}{\alpha^{15}} \Leftrightarrow \frac{a_0}{c_0} = a_0 \frac{\tau^6}{\alpha^{15}} \Leftrightarrow \tau = \sqrt[6]{\frac{\alpha^{15}}{c_0}}$$

$$\underbrace{(ab + bc + ac)}_{c_2}(k_p + R) \stackrel{s^2}{=} a_0 \frac{\tau^2}{\alpha} \Leftrightarrow (ab + bc + ac)\frac{1}{c_0} = \frac{\tau^2}{\alpha}$$

Zonk! This produces a contradiction! The equations for s^2 and s^6 are both satisfied iff

$$\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)^3 = \frac{\alpha^{12}}{abc} \Leftrightarrow \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = \frac{\alpha^4}{\sqrt[3]{abc}} \Leftrightarrow \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)^{-1} = \frac{\sqrt[3]{abc}}{\alpha^4}$$

which is usually not the case. Let us check it for a very typical case of $a = (100\pi)^2$, $b = (500\pi)^2$, $c = (700\pi)^2$ and $\alpha = 2$. This gives 9.3 vs. 6.6.

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As a consequence, the algebraic “partial”, i.e. neglecting some equations, solution provided or suggested in some papers is of little use here.

Let us test the idea in a simpler situation, i.e. with only two oscillatory terms

The controller and the plant (the open-loop transfer function):

$$G_{\text{open}}(s) = \left(k_p + \frac{sk_1}{s^2 + a} + \frac{sk_2}{s^2 + b} \right) \frac{1}{sL + R}$$

where

$$a = \omega_1^2, \; b = \omega_2^2$$

Let us reorganize this to get the characteristic polynomial of the closed-loop system

$$G_{\text{open}}(s) = \frac{k_p(s^2 + a)(s^2 + b) + sk_1(s^2 + b) + sk_2(s^2 + a)}{(s^2 + a)(s^2 + b)(sL + R)}$$

This gives

$$D_{\text{closed}}(s) = (s^2 + a)(s^2 + b)(sL + R) + k_p(s^2 + a)(s^2 + b) + sk_1(s^2 + b) + sk_2(s^2 + a)$$

Naslin polynomial

$$P_{\text{Naslin}}(s) = a_0 \left(\frac{s^5 \tau^5}{\alpha^{10}} + \frac{s^4 \tau^4}{\alpha^6} + \frac{s^3 \tau^3}{\alpha^3} + \frac{s^2 \tau^2}{\alpha} + s\tau + 1 \right)$$

Just compare coefficients to get

$$k_p \underbrace{ab}_{c_0} + Rab \stackrel{s^0}{=} a_0 \Leftrightarrow k_p + R = \frac{a_0}{c_0}$$

$$L \stackrel{s^5}{=} a_0 \frac{\tau^5}{\alpha^{10}} \Leftrightarrow a_0 = \frac{\alpha^{10} L}{\tau^5}$$

$$k_p + R \stackrel{s^4}{=} a_0 \frac{\tau^4}{\alpha^6} \Leftrightarrow \frac{a_0}{c_0} = a_0 \frac{\tau^4}{\alpha^6} \Leftrightarrow \tau^2 = \frac{\alpha^3}{\sqrt{c_0}}$$

$$\underbrace{(a+b)}_{c_2}(k_p + R) \stackrel{s^2}{=} a_0 \frac{\tau^2}{\alpha} \Leftrightarrow (a+b)\frac{1}{c_0} = \frac{\tau^2}{\alpha}$$

Zonk! This produces a contradiction! The equations for s^2 and s^4 are both satisfied iff

$$a_0 \frac{\tau^4}{\alpha^6}(a+b) = a_0 \frac{\tau^2}{\alpha} \Leftrightarrow \frac{\alpha^3}{\sqrt{ab}\alpha^6}(a+b) = \frac{1}{\alpha} \Leftrightarrow \frac{a+b}{\sqrt{ab}} = \alpha^2 \Leftrightarrow a+b = \alpha^2\sqrt{ab}$$

which is usually not the case. Let us check it for a very typical case of $a = (600\pi)^2$, $b = (1200\pi)^2$ and $\alpha = 2$. This gives 1.8 vs. 2.8. Again, it didn't work.

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As a consequence, this technique cannot be applied even for the P+R+R controller here. Sorry.

Final remarks

- Naslin polynomial method can be used for gain determination in proportional-resonant or proportional-integral-resonant controllers, but cannot be employed to tune optimally proportional-multiresonant controllers with terms of the form of $\frac{s}{s^2+a}$ xor $\frac{1}{s^2+a}$. Sorry.
- A more complex case of $\frac{k_{11}s+k_{12}}{s^2+a}$ should be investigated. I have a gut feeling it will produce similar contradiction.
- Never ever stop questioning! To question is to be scientist. To question is to be human!