On the similarity of the Naslin and Kessler approach to optimal control - the symmetrical optimum method applied to proportional-resonant controller for a grid converter

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Outline

2nd-order lag element

$$G_{\mathrm{2nd}}(s) = \frac{a_0}{a_2 s^2 + a_1 s + a_0} = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

where

$$\alpha = \frac{a_1^2}{a_0 a_2} = 4\zeta^2 \qquad \alpha = \frac{\omega_1}{\omega_0}$$

$$\omega_0 = \frac{a_0}{a_1} = \frac{\omega}{2\zeta} \qquad \omega_1 = \frac{a_1}{a_2} = 2\zeta\omega$$

is the characteristic ratio and the so-called characteristic pulsatances. $\zeta = \frac{\sqrt{2}}{2}$ is often considered as optimal trade-off between the rise time and the overshoot in process control ($\alpha = 2$).

Let us generalize this approach

$$G_{\text{Naslin}}(s) = \frac{a_0}{a_N s^N + \ldots + a_2 s^2 + a_1 s + a_0}$$

where

$$\alpha_n = \frac{a_n^2}{a_{n-1} a_{n+1}}$$
$$\omega_n = \frac{a_n}{a_{n+1}}$$
$$\alpha_n = \frac{\omega_n}{\omega_{n-1}}$$

are the characteristic ratios and the characteristic pulsatances.

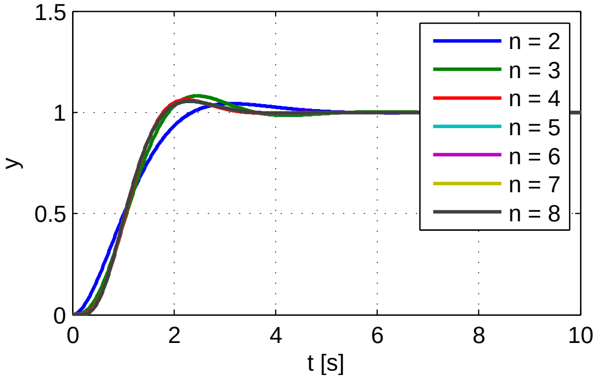
Let us now limit this generalization to $\alpha_n = \alpha$

Giving all the characteristic ratios a common value α which possesses the value of a genuine damping factor, we obtain a family of standard characteristic polynomials with adjustable damping.

$$a_0 = 1$$
$$\omega_n = \alpha^n \omega_0$$
$$\alpha_n = \omega_0^{-n} \alpha^{-n(n-1)/2}$$

What is so special about this family?

For a given value of α, the shape of the step-response does not change much with the order $N \geq 3$ and is very similar to the step response of the second order system.



Outline

The controller and the plant (the open-loop transfer function)

$$G_o(s) = \left(k_1 + \frac{sk_2}{s^2 + \omega_r^2}\right) \frac{1}{sL + R} = \frac{c_2 s^2 + c_1 s + c_0}{s^2 + \omega_r^2} \frac{1}{sL + R}$$

where

$$c_0 = k_1 \omega_r^2$$

$$c_1 = k_2$$

$$c_2 = k_1$$

The two polynomials

The characteristic polynomial of the closed loop transfer function

$$D(s) = c_2 s^2 + c_1 s + c_0 + (s^2 + \omega_r^2)(sL + R)$$

Naslin polynomial

$$P_{\text{Naslin}}(s) = a_0 \left(\frac{s^3}{\alpha^3 \omega_0^3} + \frac{s^2}{\alpha \omega_0^2} + \frac{s}{\omega_0} + 1 \right)$$

For the explanation please go to Essentials of Optimal Control (Naslin, 1969) or just google for Naslin Polynomial Method. The only design decision to be made here is selecting a suitable value for α. It is very common to set $\alpha = 2$ which means damping equal to 0.7 and thus overshoot at the level of 5%.

Just compare coefficients to get

$$c_0 + R\omega_r^2 = a_0$$

$$c_1 + L\omega_r^2 = \frac{a_0}{\omega_0}$$

$$c_2 + R = \frac{a_0}{\alpha\omega_0^2}$$

$$L = \frac{a_0}{\alpha^3\omega_0^3} \Leftrightarrow a_0 = L\alpha^3\omega_0^3$$

The resulting controller gains are

$$c_0 = L\alpha^3\omega_0^3 - R\omega_r^2$$

The proportional path gain

$$c_1 = L\alpha^3\omega_0^2 - L\omega_r^2 = k_2$$

The oscillatory path gain

$$c_2 = L\alpha^2\omega_0 - R = k_1$$

where ω_0 can be calculated from $c_0 = k_1\omega_r^2$, i.e

$$L\alpha^3\omega_0^3 - R\omega_r^2 = k_1\omega_r^2 \Leftrightarrow L\alpha^3\omega_0^3 - R\omega_r^2 = (L\alpha^2\omega_0 - R)\omega_r^2$$

which gives

$$\alpha^3\omega_0^3 = \alpha^2\omega_0\omega_r^2 \Leftrightarrow \alpha\omega_0^2 = \omega_r^2 \Leftrightarrow \omega_0 = \frac{1}{\sqrt{\alpha}}\omega_r$$

The final formula

PR gains

$$k_1 = L\omega_r \frac{\alpha^2}{\sqrt{\alpha}} - R$$

$$k_2 = L\omega_r^2(\alpha^2 - 1)$$

For an exemplary numerical experiment please see Control in Power Electronics: Selected Problems (Kazmierkowski, Blaabjerg et al., 2002).

Outline

The controller and the plant (the open-loop transfer function)

Let us assume that we would like to design *PI* controller

$$G_o(s) = k_r \frac{1 + s\tau_r}{s} \frac{k_o}{s\tau_1(1 + s\tau_2)}$$

for the plant with integration. In general, it is useless to introduce $\frac{k_o}{\tau_1}$ in the problem statement. You can substitute it with $k_o' = \frac{k_o}{\tau_1}$. Nevertheless, let us leave it like this to make it more similar to the next case study.

The two polynomials (A)

The characteristic polynomial of the closed loop transfer function

$$D(s) = s^2\tau_1(1 + s\tau_2) + k_o k_r(1 + s\tau_r) =$$
$$= s^3\tau_1\tau_2 + s^2\tau_1 + s k_o k_r \tau_r + k_o k_r$$

Naslin polynomial

$$P_{\mathrm{Naslin}}(s) = a_0\left(\frac{s^3}{\alpha^3\omega_0^3} + \frac{s^2}{\alpha\omega_0^2} + \frac{s}{\omega_0} + 1\right)$$

For the explanation please go to Essentials of Optimal Control (Naslin, 1969).

Just compare coefficients to get

$$\frac{a_0}{\alpha^3\omega_0^3} = \tau_1\tau_2$$

$$\frac{a_0}{\alpha\omega_0^2} = \tau_1$$

$$\frac{a_0}{\omega_0} = k_o k_r \tau_r$$

$$a_0 = k_o k_r$$

Let us eliminate the auxiliary variables (Naslin coefficients)

$$\frac{k_o k_r}{\alpha^3 \omega_0^3} = \tau_1 \tau_2 \Leftrightarrow k_o^2 k_r^2 = \alpha^3 \alpha^3 \omega_0^6 \tau_1^2 \tau_2^2$$

$$\frac{k_o k_r}{\alpha \omega_0^2} = \tau_1 \Leftrightarrow k_o k_r k_o^2 k_r^2 = \tau_1 \alpha^3 \omega_0^6 \tau_1^2$$

which produces

$$k_o k_r = \frac{\tau_1}{\alpha^3 \tau_2^2} \Leftrightarrow k_r = \frac{\tau_1}{\alpha^3 k_o \tau_2^2}$$

and

$$\omega_0^2 = \frac{k_o k_r}{\alpha \tau_1} = \frac{k_o \tau_1}{\alpha \tau_1 \alpha^3 k_o \tau_2^2} = \frac{1}{\alpha^4 \tau_2^2} \Leftrightarrow \omega_0 = \frac{1}{\alpha^2 \tau_2}$$

Let us now focus on τ_r

$$\frac{k_o k_r}{\omega_0} = k_o k_r \tau_r$$

Substituting

$$\omega_0 = \frac{1}{\alpha^2 \tau_2}$$

and

$$k_r = \frac{\tau_1}{\alpha^3 k_o \tau_2^2}$$

we get

$$\tau_r \frac{\tau_1}{\alpha^3 \tau_2^2} = \frac{\tau_1}{\alpha \tau_2} \Leftrightarrow \tau_r = \alpha^2 \tau_2$$

The final formula

PI optimal settings

$$k_r = \frac{\tau_1}{\alpha^3 k_o \tau_2^2}$$

$$\tau_r = \alpha^2 \tau_2$$

where again α is the only design parameter to be set. Looks familiar? Should! ;)

The controller and the plant (the open-loop transfer function)

Let us assume that we would like to design *PI* controller

$$G_o(s) = k_r \frac{1 + s\tau_r}{s} \frac{k_o}{(1 + s\tau_1)(1 + s\tau_2)}$$

for the plant with one dominant time constant

$$\tau_1 \gg \tau_2$$

where τ_2 represents a sum of small time constants.

The two polynomials (B)

The characteristic polynomial of the closed loop transfer function

$$D(s) = s(1+s\tau_1)(1+s\tau_2) + k_o k_r(1+s\tau_r) =$$
$$= s^3\tau_1\tau_2 + s^2(\tau_1+\tau_2) + s(1+k_o k_r \tau_r) + k_o k_r$$

Naslin polynomial

$$P_{\text{Naslin}}(s) = a_0\left(\frac{s^3}{\alpha^3\omega_0^3} + \frac{s^2}{\alpha\omega_0^2} + \frac{s}{\omega_0} + 1\right)$$

For the explanation please go to Essentials of Optimal Control (Naslin, 1969).

Just compare coefficients to get

$$\frac{a_0}{\alpha^3\omega_0^3} = \tau_1\tau_2$$

$$\frac{a_0}{\alpha\omega_0^2} = \tau_1 + \tau_2$$

$$\frac{a_0}{\omega_0} = 1 + k_o k_r \tau_r$$

$$a_0 = k_o k_r$$

Let us eliminate the auxiliary variables (Naslin coefficients)

$$\frac{k_o k_r}{\alpha^3\omega_0^3} = \tau_1\tau_2 \Leftrightarrow k_o^2 k_r^2 = \alpha^3\alpha^3\omega_0^6\tau_1^2\tau_2^2$$

$$\frac{k_o k_r}{\alpha\omega_0^2} = \tau_1 + \tau_2 \underset{\tau_1 \gg \tau_2}{\Rightarrow} \frac{k_o k_r}{\alpha\omega_0^2} = \tau_1 \Leftrightarrow k_o k_r k_o^2 k_r^2 = \tau_1\alpha^3\omega_0^6\tau_1^2$$

which produces

$$k_o k_r = \frac{\tau_1}{\alpha^3\tau_2^2} \Leftrightarrow k_r = \frac{\tau_1}{\alpha^3 k_o \tau_2^2}$$

and

$$\omega_0^2 = \frac{k_o k_r}{\alpha\tau_1} = \frac{k_o\tau_1}{\alpha\tau_1\alpha^3 k_o\tau_2^2} = \frac{1}{\alpha^4\tau_2^2} \Leftrightarrow \omega_0 = \frac{1}{\alpha^2\tau_2}$$

Let us now focus on τ_r

$$\frac{k_o k_r}{\omega_0} = 1 + k_o k_r \tau_r$$

Substituting

$$\omega_0 = \frac{1}{\alpha^2 \tau_2}$$

and

$$k_r = \frac{\tau_1}{\alpha^3 k_o \tau_2^2}$$

we get

$$1 + \tau_r \frac{\tau_1}{\alpha^3 \tau_2^2} = \frac{\tau_1}{\alpha \tau_2} \underset{\tau_1 \gg \tau_2 \wedge \alpha \approx 2}{\Rightarrow} \tau_r \frac{\tau_1}{\alpha^3 \tau_2^2} = \frac{\tau_1}{\alpha \tau_2} \Leftrightarrow \tau_r = \alpha^2 \tau_2$$

The final formula

PI optimal settings

$$k_r = \frac{\tau_1}{\alpha^3 k_o \tau_2^2}$$

$$\tau_r = \alpha^2 \tau_2$$

where again α is the only design parameter to be set. Looks familiar? Should! ;)

Outline

Kessler symmetrical optimum for PI and one dominant time constant

PI optimal settings

$$k_r = \frac{\tau_1}{8 k_o \tau_2^2}$$

$$\tau_r = 4\tau_2$$

is equivalent to the previous formula with $\alpha = 2$.

Final remarks

- Naslin polynomial method can be presented as the generalization of the symmetrical optimum method.
- The names authors use to label methods often shape our understanding of the solution. Many authors use Naslin polynomial to tune PR controller but almost none of them refers to SO. Why?
- Because it is a cultural thing. The method is known as Kessler's SO mainly in Germany and Poland. In US and GB they can get confused if you refer to SO but they are familiar with Naslin method.
- If your goal is to make the speech ambiguous it is recommended to fill it with names, e.g. Mendeleev's table instead of the periodic table and so on and so forth.